Pulse confinement in optical fibers with random dispersion

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Short range correlated uniform noise in the dispersion coefficient, inherent in many types of optical fibers, broadens and eventually destroys all initially ultra-short pulses. However, under the constraint that the integral of the random component of the dispersion coefficient is set to zero, or pinned, periodically or quasi-periodically along the fiber, the nature of the pulse propagation changes dramatically. For the case that randomness is added to constant positive dispersion, the pinning restriction significantly reduces pulse broadening. If the randomness is added to piecewise constant periodic dispersion, the pinning may even provide probability distributions of pulse parameters that are numerically indistinguishable from the statistically steady case. The pinning method can be used to both manufacture better fibers and upgrade existing fiber links.

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The effect of random perturbations in optical fibers increasingly attracts attention as the demand for the quality of transmission grows daily [1]. The impact of randomness on signal transmission in a single mode fiber is negative; it causes degradation of the signal and lowers transition capabilities [2]. In particular, amplifier noise [3], and noise in the fiber birefringence (double refraction) [4] lead to random shifts in the pulse position (timing jitter) and to pulse broadening, respectively. Both effects eventually cause a destruction of bit-patterns and eventual increase of the Bit-Error-Rate (BER), the most important parameter describing performance in fiber communications systems [5].

In the present paper, we consider the effect of random dispersion, which is, for ultrashort pulses, potentially as dangerous as the aforementioned effects. We, however, propose a way to significantly reduce the pulse deterioration and eventually reduce the BER caused by the noise in dispersion by using passive (independent of pulse properties) periodic control of the accumulated dispersion of the fiber link. Furthermore, the method may even provide statistically steady propagation of the pulse along the fiber.

Chromatic dispersion is an important characteristic of a medium and can significantly degrade the integrity of wave packets. In practice, chromatic dispersion is not uniformly distributed and often exhibits random variations in space and time. On the other hand, wave propagation through the medium is usually much faster than temporal variations of the chromatic dispersion. Therefore, these random variations can be treated as "spatial" multiplicative noise that does not change in time. This multiplicative noise is conservative and the wave energy remains constant during propagation through the medium. Recently, high precision measurements of fiber chromatic dispersion as function of a fiber length experimentally demonstrated the significance of the dispersion randomness [6,7].

The overall chromatic dispersion in an optical fiber comes from two sources. The first source is the medium itself. The second source is the specific geometry of the waveguide profile. Material dispersion in the optical fiber is a relatively stable parameter, uniformly distributed along the fiber. However, waveguide dispersion is not nearly as stable. Existing technology does not yet provide accurate control of the waveguide geometry of modern fibers, where dependence of dispersion on wavelength is complex. As a result, the magnitudes of random variations of fiber chromatic dispersion are typically the same as, or in some cases even greater than that of the mean dispersion [6,7].

In the short-wavelength regime, a universal description of the signal envelope in the reference frame moving with the packet group velocity is given by the nonlinear Schrödinger equation (NLS) for the complex scalar field, $\psi(z;t)$, see for example [5],

$$-i\partial_z\psi = d(z)\partial_t^2\psi + 2\psi^2\bar{\psi}. \tag{1}$$

The equation is written in the dimensionless form [8]. Variations in the medium enter this description through the dispersion coefficient $d(z) = d_{\rm det}(z) + \xi(z)$, which is decomposed into its deterministic part, $d_{\rm det}(z)$, and a random part, $\xi(z)$. Here z is the position along the fiber and t is the retarded time. The initial profile $\psi(0;t)$ is localized in t. We consider two different models of deterministic dispersion, both of which are standard in fiber optic communications. Model (A) is the case of constant dispersion, $d_{\rm det} = d_0$. In the absence of noise ($\xi(z) = 0$), $\psi_0(z;t) = a\exp\left[izd_0/b^2\right] sech\left[t/b\right]$, where $a^2b^2 = d_0$ (a is the peak amplitude, b is the pulse width), is an exact soliton solution of (1). The existence of the soliton [9] is the result of a dynamic equilibrium between dispersion and nonlinearity: the two spatial scales, nonlinearity $z_{NL} = 1/a^2$, and dispersion $z_d = b^2/d_0$, coincide. Model (B) is the case of dispersion management (DM), $d_{\rm det} = d_0 \pm d_{DM}$ [10]. Here dispersion is piecewise constant: positive and negative spans alternate with period

z_{DM}, and $0 < d_0 < d_{DM}$. There is no exact solution for the pure (no noise) Model (B), but theoretical evidence, confirmed by extensive numerical studies and experimental results, indicates the existence of a breathing solution (DM soliton) with a nearly Gaussian shape [11–14]. The localized solution here is again due to the interplay of dispersion and nonlinearity. In the presence of a periodic dispersion map, however, the (DM) soliton acquires an important characteristic, quadratic phase (chirp). In contrast to conventional soliton solutions, DM solitons can exist for zero (or even negative) values of average dispersion.

Approximate scale characteristics of the dispersion noise present in real fibers can be extracted from experimental results [6,7]. These results show that the smallest scale of noticeable change in the dispersion value is approximately $\sim 1-2km$. For constant dispersion fibers (model A), the amplifier spacing is $\sim 50-60km$, and for dispersion managed fibers (model B), the period of a typical dispersion map is also $\sim 50-60km$. These scales are much longer than that of the dispersion variation, justifying the idealized consideration of delta-correlated noise for both models. Previously, the stability of initial pulses in the presence of the short-range-correlated Gaussian uniform noise ξ_u with zero mean, $\langle\xi_u(z_1)\xi_u(z_2)\rangle = D\delta(z_1-z_2)$, was studied for both models (A) and (B). For model (A), adiabatic theory, valid if the noise is weak, shows dynamical broadening of the pulse and its eventual destruction [15]. The pulse-broadening effects of ξ_u on model (B) were studied numerically and by means of a variational approach [15,16].

The problem can also be addressed in the limit of strong noise. On short scales the nonlinearity is weak and propagation is essentially linear : only the phase of the pulse is changed by a rapidly varying dispersion, while its frequency spectrum is not. In this weakly nonlinear case the following change of variables is suggested, $\psi(z;t) = \int_{-\infty}^{\infty} d\omega \exp\left[-i\left(\omega t + \omega^2\left[\int_0^z (d(z')-d_0)\,dz'\right]\right)\right]\psi_\omega(z)$, where $\psi_\omega(z)$ describes the pulse evolution on longer scales. The equation for the noise average of the slow filed $\varphi_\omega = \langle\psi_\omega(z)\rangle$ derived from (1) is

$$\left(-i\partial_z + id_0\omega^2\right)\varphi_\omega = 2\int d\omega_{1,2,3}\delta\left(\omega_1+\omega_2-\omega_3-\omega\right)$$
$$\times\exp\left[-i\Delta\int_0^z dz'\left(d_{\rm det}-d_0\right)\right]\, e^{-\Delta^2 D z_*/2}\;\varphi_1\varphi_2\bar{\varphi}_3. \quad (2)$$

Here, $\Delta \equiv \omega_1^2+\omega_2^2-\omega_3^2-\omega^2$, and the second exponential on the right of (2), which we will call the kernel, is the average of $\exp\left[-i\Delta\int_0^z \xi(z')\,dz'\right]$. $z_* = z$, thereby setting the correlation scale, $z_\xi = [\Delta^2 D]^{-1} \sim [\omega^4 D]^{-1} \sim b^4/D$. The approximation is justified, i.e. the nonlinearity is weak if $z_\xi \ll z_{NL}$. The exponential decay of the kernel with z disrupts the balance between nonlinearity and dispersion necessary for steady pulse propagation.

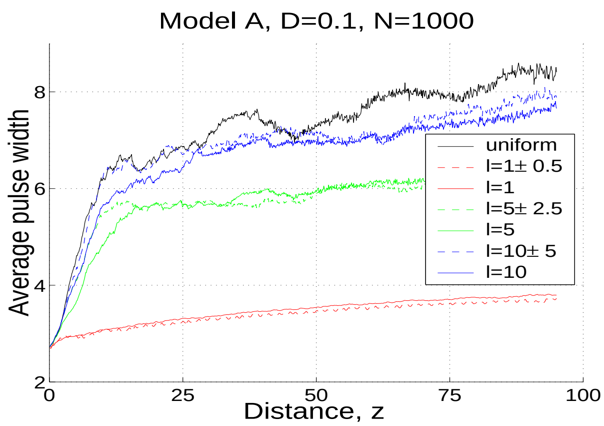
The grim analytical conclusion is that the natural noise in dispersion leads to destruction of initially localized signal in the extremes of both weak and strong noise. Numerical study of the intermediate range shows the same effect.

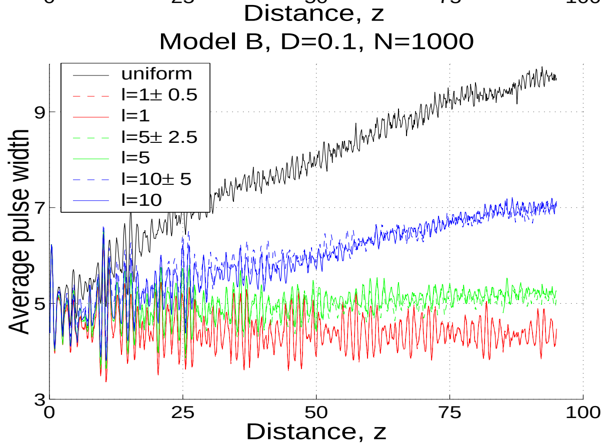
The natural question is that of the existence of *an artificial constraint* which may reduce or completely prevent this pulse broadening. We demonstrate that such a constraint does indeed exist, and can be readily implemented in real fibers. All that is required is that *the accumulated dispersion,* $\int_0^z dy\xi(y)$*, is set to zero, or pinned, either periodically or quasi-periodically* with a period of the order or less than z_ξ. The resulting Gaussian nonuniform noise, ξ_n with zero mean is described by $\langle\xi_n(y)\xi_n(z)\rangle = D\left(\delta(z-y) - \frac{1}{l_{j+1}-l_j}\right)$ if y and z belong to the same segment bounded by an adjacent pair of pinning points. Otherwise there are no correlations.

Consider the effect of nonuniform noise, $\xi = \xi_n$, in the weakly nonlinear case. z_* in (2) is $z\left(l_{j+1}-z\right)/\left(l_{j+1}-l_j\right)$ with $z \in [l_j; l_{j+1}]$. Thus the decay of the kernel in (2) is replaced by oscillatory (or quasi-oscillatory, when the period is fluctuating) behavior. In the nonuniform case, additional averaging over a single pinning leg reduces the z-dependent kernel of (2) to $\exp\left[-\Delta^2 Dl/4\right]\sqrt{\pi/[\Delta^2 Dl]}\mathrm{Erfi}\left[\sqrt{\Delta^2 Dl}/4\right]$. In Model (B), independent averaging of the first exponential on the right-hand side of (2) over z_{DM} (see also [11,12]) replaces it by $2\sin\left[\Delta z_{DM}/4\right]/\left[\Delta z_{DM}\right]$. The overall result of the averaging procedure in the case of $\xi = \xi_n$ is the emergence of a nonlinear term which does not vanish as $z \to \infty$, in contrast to the case $\xi = \xi_u$, where the nonlinearity dies away with $z \to \infty$. One also concludes that z_ξ sets the critical pinning period: the pinning is efficient only if $l = l_{i+1} - l_i \lesssim z_\xi$. It is possible to show that the averaged equation for $\xi = \xi_n$ has a steady solution [17]. Notice also that the averaged kernel decays exponentially as $\Delta^2 \to \infty$. This is much faster than the algebraic decay obtained from averaging over the dispersion map period in pure dispersion management. Therefore, the model of [18] corresponding to a very narrow type of nonlinear kernel (in ω-space) is better suited for the case considered here than for the case of pure DM for which it was originally proposed. The averaged equation approach is *á priori* applicable for a large but finite z, so the emergence of a steady localized solution for the averaged equation does not mean that the original problem possesses a steady state. In the direct numerical simulation of (1), to which we now switch our attention, the effect is seen as essentially limiting the pulse broadening.

We perform numerical investigations of both models (A) and (B) with $\xi = \xi_u$ and $\xi = \xi_n$ in intermediate case, $z_{NL} \sim z_\xi$. Fourier split-step scheme with 2^{13} temporal Fourier modes and periodic conditions imposed on the boundaries of the domain $t \in [-180, 180]$ is implemented. The spatial step is $z_{step} = 0.01$ and the numer-

ical convergence was checked by varying the size of the periodic box and number of the Fourier harmonics. Parameters for the initial signal were chosen to be $d_0 = 1$, $a = 1$ in model (A), and $d_0 = 0.15$, $d_{DM} = 0.1$, $z_{DM} = 1$, $|\psi(0;t)| = 0.79\exp(-t^2/2.6)$ in model (B). The setup in model (B) is borrowed from [13] and corresponds to experimentally available DM fibers. Gaussian zero-mean noise correlated at $z_n = 0.1$ with amplitude $d_n = 1$ models the δ-correlated uniform noise with $D = d_n^2 z_n = 0.1$. The nonuniform noise is constructed from the uniform noise by the following subtraction at every pinning leg: $\xi_n(z) = \xi_u(z) - \frac{1}{l_{j+1}-l_j}\int_{l_j}^{l_{j+1}} dy\xi(y)$. Pinning strategies of two types are considered: strictly periodic, $l_{j+1} - l_j = l$, where l is fixed; and quasi-periodic, $l_{j+1} - l_j = l(1+\eta)$, where η is a random number uniformly distributed between $\pm 1/2$. The averaged (or otherwise strict) pinning period for the nonuniform case is taken to be 1, 5 or 10. The simulation runs until the pulse arrives at $z = 95$. Statistics were collected for $10^2 - 10^3$, and in a special case 10^4, realizations.





The averaged pulse-width (full width at half maximum amplitude) as a function of z is shown in Figure 1, see also [19]. The capital letter subscript of the figures corresponds to the type of model, (A) or (B). Solid black, red, green, and blue represent uniform noise, and nonuniform noise with pinning period $l = 1, 5, 10$ respectively. The quasi-periodic curves are dashed and of the same color as the respective periodic ones.

For Model(A), all types of nonuniform noise demonstrate a significant reduction in the rate of pulse broadening when compared with the uniform case. The individual configurations that degrade (through pulse splitting, etc.) in the uniform case maintain pulse integrity when each type of nonuniform compensation is applied. The dependence on the pinning period is monotonic: the peak amplitude of the pulse decays faster as the pinning period increases. The difference between periodic and respective quasi-periodic cases is minor, with a slightly better confinement observed for the quasi-periodic case. The destruction of the pulse is also accompanied by emission of continuous radiation by the soliton. The radiation is clearly seen in the movie made for individual runs [19]. Once the radiation reaches the boundaries of the box, it reflects and starts to interfere with the still localized solution. The latter shows up in the change of the averaged-width behavior at larger distances, $z \sim 20$. In principle, one could introduce absorbing boundary conditions to enable longer-time simulations. However, these conditions are artificial, as in real data streams pulses are not isolated. Each pulse emits continuous radiation which eventually interacts with neighboring pulses. In this paper we purposefully study self-interaction of a pulse and its radiation via reflecting boundary conditions, which is a simple model for the behavior of a real bit stream. Our results reveal that after $z \sim 20$, the rate of pulse disintegration is reduced, suggesting that in a real bit pattern, pulse disintegration might be prevented by radiation absorption from neighboring pulses.

The effect of nonuniform noise is more dramatic in the case of Model(B). For nonuniform compensation with the averaged period $l = 1$ (and also less) one observes a tendency toward statistically steady behavior: the average pulse width does not decay (in contrast to a decay in the uniform case), and the PDF of the pulse width (and of other variables characterizing the pulse propagation, such as amplitude) does not change shape with z [20]. There is no visible emission of radiation by the localized solution for any case of Model (B). We have also checked that temporal and spatial averages (for example for the PDF of the pulse width) coincide. The dependence on the type of compensation (for $l > 1$) is monotonic, and the difference between the random and quasi-random cases is minor, as in the case of Model(A).

Notice that the tremendous reduction in the pulse decay is achieved by very minor changes in dispersion (see [19], for the comparable plots of the dispersion profiles).

Our analysis gives practical recommendations for improving fiber system performance that is limited by randomness in chromatic dispersion. The limitation originates from accumulation of the integral dispersion. The distance between naturally occurring nearest zeros of the accumulated dispersion grows with the fiber length ($\sim \sqrt{z}$ as it is shown in [17]). This growth causes asymp-

totic decay of the nonlinearity with z and, therefore, pulse degradation. We have shown that the signal can be stabilized by periodic or quasi-periodic pinning of the accumulated random dispersion [21]. This can be achieved by first measuring the mismatch between the nominal dispersion of the fiber line and the actual accumulated dispersion, and second, inserting a small fiber to compensate this mismatch [22].

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[19] Movies of single realization dynamics in z, comparative plot of the dispersion profile with and without compensation, and more figures characterizing statistics of the pulse propagation (in the various cases considered) are available at http:/cnls.lanl.gov/˜chertkov/Fiber.

[20] Notice that for the case with the same pinning period $l = 1$ but greater $D = 2.5$, a minor, but still observable degradation of pulse occurred. This is consistent with the statement concerning the efficiency of the pinning made in the text: the greater D, the lower the critical pinning period.

[21] Real fiber spans, usually each of length $0.5 - 2\ z_{NL}$, are not homogeneous, as they are often produced at different times by different companies.

[22] After the work was completed we learned about the paper of L.F.Mollenauer, et al. [Opt. Lett. **25**, 704 (2000)], where long haul transmission experiments on fibers built from spans of different types are described. It was shown there that periodic compensation of the overall dispersion to the fixed residual value (achieved via insertion of an extra span) optimizes propagation of pulses. These results are consistent with, and also give an experimental support to, the present proposed pinning method.